ILLUSTRATION OF THE ARCH.

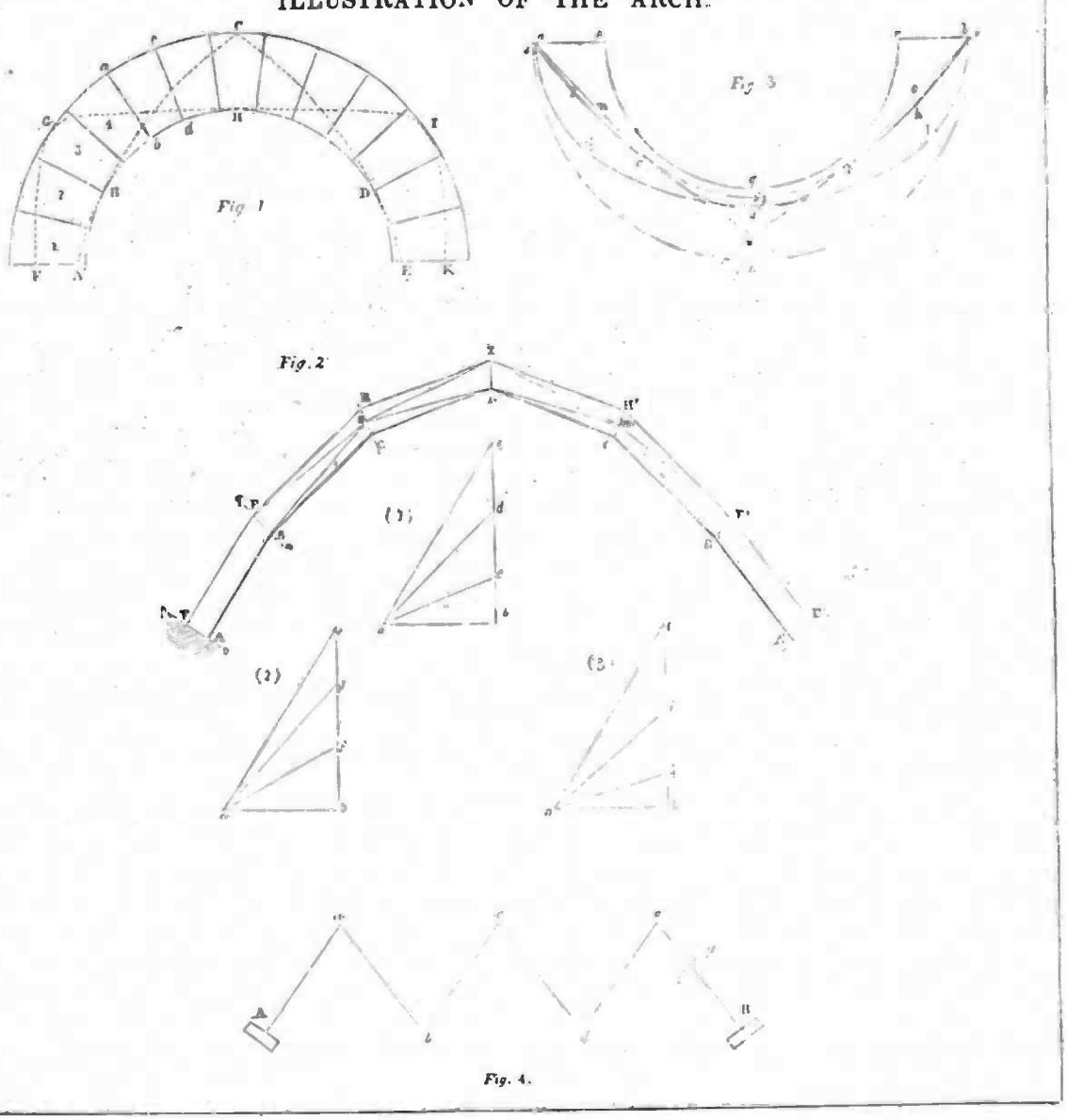

Fig. 1

Fig. 3

Fig. 2

Fig. 4.

was loaded in the proportions above represented, the frame would be in equilibrium, even though friction did not act.

As no material is in reality perfectly smooth, so the pieces in the frame would in practice require some force to cause them to slide on each other at the bed-joints, and the friction, thus induced, would tend to preserve the equilibrium. If the resistance from friction at the bed-joints be taken into account, the proportions of weight may vary from that exhibited above within certain limits; these limits are defined by the magnitude of the angle of sliding; that is, the angle at which the pieces in the polygon would begin to slide upon each other.

To demonstrate this, let the proportion of weight be altered from that exhibited in (1), and instead, put the increased weight *bf*, (2), at the point I, *fg* at H, and *ge* at F, draw the lines *ab*, *af*, *ag*, and *ae*, and they will be the geometrical representatives of the magnitude and direction of the pressures on the bed-joints. Parallel to these lines, and within (if possible), the boundaries of the polygonal frame, draw the line 1*m*, *m*B, BA, and the curve 1*m*BA, will represent the line of pressure in one-half of the frame; and the angles at which this line of pressure intersects the bed-joints determines the stability of the structure; for so long as the angles 1*m*H, *m*B*q*, B*Ap*, are greater than the complement of the angle at which the pieces would begin to slide on each other, the structure is safe; but if 1*m*H, *m*B*q*, or B*Ap*, is less than the complement of the angle of sliding, then the pieces IC, HB, or FA, would begin to slide in the direction HC, FB, or EA; also if *m*ID is less than the complement of the limiting angle of friction, CI would slide upwards in the direction DI, so that on either of these suppositions, the polygon would be destroyed.

Also within the same limits the proportion of the load may vary by loading the haunches and reducing the weights on the crown, for let *bh* (3) be less than *bc* (1), and *hi* greater than *ed*. Then *bh* will be the weight at I, *hi* the weight at H, and *ie* the weight at F. The haunches of the frame are now loaded with a greater weight in proportion to the crown than they are in (1). From *a* draw *ah*, *ai*, *ae*, and parallel to them, within the polygon, draw the lines D*m*, *m*F and FE, and D*m* FE will be the line of pressure; and D*m*, C*m*, B*n*, and BA*o* will be less than the limiting angle of resistance, beyond which, if either go, the piece would slide on its adjacent bed-joint. Also, if *m*DI is less than the complement of the angle of sliding, the piece H would slide downwards in the direction ID. By the foregoing reasoning, it is shewn that if an arch be loaded at the crown, the line of pressure rises at the crown and descends at the haunches, and if the arch be loaded at the haunches, the line of pressure rises at the haunches and descends at the crown, so that if the load were on the crown, the line of pressure would be ABCDE, and if it were on the haunches it would be FGHIK. In order that in neither of these cases the voussoirs may slide on each other, the line of pressure must not at any point make an angle with either of the bed-joints less than the complement of the angle of sliding.

The second condition of stability is fulfilled so long as the line of pressure does not intersect either the extrados or intrados. When the crown is loaded, we see that the point nearest the extrados is C, and the points nearest the intrados are B and D, and, by increasing the weight, the line of pressure would intersect the boundaries of the voussoirs at one or more of these points, when it did, the consequence would be that the crown would descend and the haunches ascend the arch, being thereby destroyed. Again, if the haunches are loaded, the line of pressure approaches nearest the boundaries at the points GH and I, and if the load is increased the arch would yield at one or more of these points, when the crown would ascend and the haunches descend. These are briefly the geometrical principles of the two conditions necessary for the stability of the arch, from which we will endeavour to deduce some practical observations.

By examining fig. 1, it will be seen that if the arch is intended to support a heavy weight on the vertex, it should be of the *Gothic* or *pointed* form. If it be designed to sustain a load on its haunches, the form of the *ellipse* is

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